CURRICULUM, PEDAGOGY AND BEYOND





MAV division





CONNECTING TEACHING TO THE NATURE OF MATHEMATICS TO BE LEARNED Subtheme: Pedagogy Peter Sullivan, Monash University (F to Year 10)

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Abstract

- Planning, teaching, learning and assessment are ideally informed by the nature of mathematics in which explicit emphasis is on connections between ideas, flexible thinking, application of mathematics to practical contexts, and ways that problem solving and reasoning connect to understanding and fluency.
- This session will illustrate the characteristics of such teaching (in the context of VTLM 2.0) and will contrast this with approaches that reduce mathematics to routines and rules to be learned by rote.

Victorian Teaching and Learning Model 2.0

Elements of learning

Attention, focus and regulation

Refers to learning requiring students' attention and involving active engagement in a supportive and responsive learningfocused environment.

Knowledge and memory

Refers to students

information in their

where they connect

knowledge in long-

models that integrate

working memory,

it with existing

term memory,

and organise

knowledge.

building mental

processing new

کر (Retention and recall

Refers to working memory being able to hold a small amount of information at once (cognitive load). If overloaded, new knowledge won't be effectively stored in long-term memory.



Mastery and application

Refers to consistent practice and retrieval, allowing students to develop and demonstrate mastery by retaining knowledge and understanding how to apply it effectively.

- I assume you emphasise Attention ... and Mastery ... already.
- These two require attention.

{ ငို Knowledge and memory

Refers to students processing new information in their working memory, where they connect it with existing knowledge in longterm memory, building mental models that integrate and organise knowledge. MAV division



Refers to working memory being able to hold a small amount of information at once (cognitive load). If overloaded, new knowledge won't be effectively stored in long-term memory.

Elements of teaching

Refers to the collaborative development of whole school teaching and Planning learning programs that break down and sequence the knowledge to be taught and assessed. It also refers to the planning required to implement the curriculum into the classroom and to the school-wide enactment of a multitiered system of supports. Refers to the positive relationships, cultural responsiveness, classroom Enabling expectations and management techniques that teachers establish and use learning to foster student self-regulation and self-efficacy, and to create a learningfocused environment where the development and application of knowledge drives curiosity and creativity. Refers to the evidence-based practices that manage the cognitive load Explicit of students, including activating prior knowledge, clearly stating learning teaching objectives, providing explicit explanations of new knowledge, scaffolding learning and modelling practice, and using formative assessment and feedback to monitor progress towards mastery. Supported Refers to the practices that maximise the consolidation and application of application

learning, including revisiting and reviewing knowledge, varying and spacing practice, organising knowledge and extending and challenging students as they move to mastery of new factual, conceptual and procedural knowledge.

- Planning, Enabling Learning and Supported Application are familiar to everyone.
- Explicit Teaching is potentially different.

Explicit teaching Refers to the evidence-based practices that manage the cognitive load of students, including activating prior knowledge, clearly stating learning objectives, providing explicit explanations of new knowledge, scaffolding learning and modelling practice, and using formative assessment and feedback to monitor progress towards mastery.

VTLM 2.0 - Explicit teaching and cognitive load

- What does explicit teaching look like (and what is it not)?
- How is cognitive load managed, recognising that the nature of mathematics includes
 - building connections,
 - fostering intuitive and adaptable thinking, and
 - using mathematics (numeracy) to do something else?

Cognitive load and relational mathematics learning



Schema

- Well constructed knowledge is linked together so that when one part of a network of ideas is recalled for use at some future time, the other parts are also recalled.
- The key is developing connections between ideas (which is central to mathematical thinking)

Building understanding relationally

- Students and parents tend to over emphasise the "HOW" of mathematics and numeracy (instrumental understanding)
- But we want students to come to understand both "WHY" and "HOW" (relational understanding)



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Suppose we want Grade 5 students to imagine problems involving division

- multiply and divide one- and two-digit numbers, representing problems using number sentences, diagrams and arrays, and using a variety of calculation strategies VC2M3N05
- develop efficient mental and written strategies and use appropriate digital tools for solving problems involving addition and subtraction, and multiplication and division where there is no remainder VC2M4N06
- solve problems involving division, choosing efficient mental and written strategies and using digital tools where appropriate; interpret any remainder according to the context and express results as a whole number, decimal or fraction (VC2M5N07)



Work out the answer to this question, explaining your reasoning using equations:

We need to order buses to take all the students in the school to a concert.

There are 435 students and each bus can take 20 students.

How many buses do we need to order?

- What would you "tell" the students before posing this question?
- What might make the question difficult for (some) students?

Of course, we could have used the numbers 1144 and 32

Work out the answers to this question, writing your answer as an equation, and explain your reasoning:

- We need to book buses to take all the students in the school to a concert.
- There are 1144 students and each bus can take 32 students.
- How many buses do we need to order?

Addressing (extraneous) cognitive load without limiting agency and engagement

- Clarify language
- Check for interpretation of representations
- Address potential misconceptions explicitly
- Revise and practice relevant pre-requisite fluency
- Explain expectations for the task
- Present and clarify Learning Intentions (what, why, how)



What are the important words in this question?

There are 16 students wanting to play basketball. I decide to make 2 teams. How many students in each team?



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Representations

Draw a picture to show how you work out the answer Show your answer as an equation

There are 16 students wanting to play basketball.

- I decide to make 2 teams.
- How many students in each team?

Misconceptions

Which of the following shows how to record an equation to show how you worked out the answer?



Relevant Fluency

• Clap counting

× 2 × 10 × 20

•••

Relevant Fluency

double ... Groups of 2 ... Groups of 10 ... Groups of 20

• • •

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Relevant Fluency

20	
40	
30	
50	
70	
80	
60	
90	

Double Double double

Expectations

- show how you get your answers including explaining your thinking
- the numbers are the same but the answers are different
- try to imagine what the question is asking
- it might help to draw a picture

The goal for your learning

By imagining what the question is asking I am learning to use division to solve problems to help me learn to use multiplication and division in the future

If you are stuck



If we had 85 students, and

20 students can travel on each bus,

how many buses would we need?

If you have finished

How would you do it if these are the numbers?

We need to order buses to take all the students in the school to a concert.

There are 439 students and each bus can take 23 students.

How many buses do we need to order?

435 and 20

Work out the answers to these questions, show your working using equations, explaining your reasoning (the answers are all different):

- We need to book buses to take all the students in the school to a concert. There are 435 students and each bus can take 20 students. How many buses do we need to order?
- We are making up packets of chocolates. Each packet must have exactly 20 chocolates. If I have 435 chocolates, how many complete packets can we make?
- Our basketball club won a prize of \$435. The 20 members decided to share the prize exactly between them. How much money will each of them get?

435 and 20

Work out the answers to these questions, writing your answer as an equation, and explain your reasoning (the answers are all different):

- We need to book buses to take all the students in the school to a concert. There are 435 students and each bus can take 20 students. How many buses do we need to order?
- We are making up packets of chocolates. Each packet must have exactly 20 chocolates. If I have 435 chocolates, how many complete packets can we make?
- Our basketball club won a prize of \$435. The 20 members decided to share the prize exactly between them. How much money will each of them get?

In what ways are the questions similar and in what ways are they different?

More 435 and 20

Work out the answers to these questions, writing your answer as equations, and explaining your reasoning (the answers are all different from each other and from the ones before):

- The train from Melbourne to Sydney travels at an average speed of 20 km/hr. How long would it take to travel the 435 km to Sydney if the train does not stop?
- Our year level of 20 students together won a prize of 435 pizzas If we share the prize equally, how much pizza do we each get?
- A dairy farm produced 435 litres of milk, and has 20 containers in which to store the milk. If the containers are filled exactly, how much milk should go into each container?

In what ways are the questions similar and in what ways are they different?

How many are left?

There are 435 people who need to cross a crocodile infested river.

The ferry can carry 20 people each trip.

If everyone is in a hurry to cross the river, how many people will be left for the last trip?

74÷6

Work out the answers to these questions, writing your answer as an equation, and explain your reasoning (the answers are all different):

- A farmer needs to take 74 pigs to the market. 6 pigs can fit into his trailer. How many trips does he need to make?
- Our year level has 74 students. We want to make teams of 6 for volleyball. How many teams can we make?
- I can write 6 birthday invitations in an hour. How long will it take me to write 74 invitations?
- I have 74 cm of ribbon. I want to make 6 strips of ribbon for decorations. How long will be each strip of ribbon?

In what ways are the questions similar and in what ways are they different?

Using a calculator

How could you use a calculator to work out the remainder to 4783 ÷ 76?

l can ...

- Imagine the meaning of a problem written in words
- Write equations to match a given word problem
- Match the answer to word problems to the meaning of the question
- Explain my thinking

Another example







From the Victorian Curriculum

- <u>VC2M7N09</u>
- recognise, represent and solve problems involving ratios
- <u>VC2M8M05</u>
- recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure
- <u>VC2M8M07</u>
- use mathematical modelling to solve practical problems involving ratios and rates, including distance-time problems for travel at a constant speed and financial contexts; formulate problems; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model

How many cups?

The recipe for 4 people suggests 2 ½ cups of vegetable stock.

- How many cups of stock do I need to make soup for 18 people?
- Work this out two different ways



In anticipation ...

- What do we hope students will learn/do (assuming we have not told the students a method for solving the task)?
- What might students find difficult or barriers to their learning?

An intuitive method

- 4 people use 2 ½ cups
- 8 people use 5 cups
- 16 people use 10 cups
- 2 people use 1 ¼ cups

The unitary method

```
4 people need 2\frac{1}{2} cups
1 person would need \frac{1}{4} of 2\frac{1}{2} cups
= \frac{1}{4} \times \frac{5}{2}= \frac{5}{8}
18 people need 18 \times \frac{5}{9}
=\frac{90}{8}= 11\frac{2}{8} = 11\frac{1}{4}
```

A formal method



How many cups?

The recipe for 4 people suggests $2\frac{1}{3}$ cups of vegetable stock.

- How many cups of stock do I need to make soup for 18 people?
- Work this out two different ways



An instructional model





MAV division



How many cups?

The recipe for 4 people suggests 2 ½ cups of vegetable stock.

- How many cups of stock do I need to make soup for 18 people?
- Work this out two different ways



Finding a unit

```
4 people need 2\frac{1}{2} cups
1 person would need \frac{1}{4} of 2\frac{1}{2} cups
= \frac{1}{4} \times \frac{5}{2}= \frac{5}{8}
18 people need 18 \times \frac{5}{2}
=\frac{90}{8}= 11\frac{2}{8} = 11\frac{1}{4}
```

In this case, it would be easier as

```
4 people need 2\frac{1}{2} cups
2 people would need \frac{1}{2} of 2\frac{1}{2} cups
=1\frac{1}{4}
18 people need 9 \times \frac{5}{4}
= <u>45</u>
= 11 \frac{1}{1}
```

A formal method



An instructional model



Enabling prompts (for students who find it too difficult)

- It may be that the use of fractions makes the proportional thinking difficult
- In general, we can create enabling prompts by:
 - Simplifying the numbers
 - Changing the representation
 - Reducing the number of steps

If you are stuck

The recipe for 4 people suggests 2 cups of vegetable stock. How many cups of stock do I need to make soup for 10 people?

If you have finished

- Will your methods work whatever are the fractions?
- For example:

The recipe for 5 people suggests 2 $\frac{3}{4}$ cups of vegetable stock. How many cups of stock do I need to make soup for 18 people? Work this out two different ways



An instructional model





It takes 2 minutes to fill $\frac{3}{4}$ of a bucket. How long would it take to fill 9 buckets?



Show two different ways to find the answer.



It takes 2 minutes to fill $\frac{3}{4}$ of a bucket. How many buckets can I fill in 15 minutes?

Show two different ways to find the answer.



I can ...

- Use multiplication (and division) of fractions to solve a rate problem
- Use intuitive methods to solve a rate problem
- Use the unitary method to solve a rate problem
- Use formal methods to solve a rate problem
- Explain my reasoning in writing
- Explain my reasoning verbally





Event App

App Download Instructions

Step 1: Download the App 'Arinex One' from the App Store or Google Play



- Step 2: Enter Event Code: mav
- Step 3: Enter the email you registered with
- Step 4: Enter the Passcode you receive via email and click 'Verify'. Please be sure to check your Junk Mail for the email, or see the Registration Desk if you require further assistance.

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Be in it to WIN!

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A02 - (Year 1 to Year 6) Supporting High Potential and Gifted Learners in Mathematics

☆ Add to Favourite > Complete the Survey > (i) Description

ମ୍ ≣ Speaker



Dr Chrissy Monteleone

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- Peter and Paul agreed to host a party and share the costs.
- Peter spent \$230 on food.
- Paul spent \$120 on dirnks.
- How do they share the costs?

